Let $G$ be a finite graph with vertex set $V$ which may not necessarily be loopless. For each $v \in V$, let $\alpha_v \in \mathbf{F}_2^V$ be the map which takes value 1 on $w \in V$ if and only if there is an edge between $v$ and $w$ in $G$; let $\mathcal{T}_v$ be the linear map on $\mathbf{F}_2^V$ that sends $x \in \mathbf{F}_2^V$ to itself if $x(v) = 0$ and sends $x \in \mathbf{F}_2^V$ to $x + \alpha_v$ if $x(v) = 1$. Note that $\mathcal{T}_v$ is a transvection when $v$ is not a loop in $G$ while $\mathcal{T}_v$ is an idempotent when $v$ is a loop in $G$. We consider the digraph $\Gamma$ with vertex set $\mathbf{F}_2^V$ and arc set $\{(x, \mathcal{T}_v(x) : x \in \mathbf{F}_2^V, v \in V\}$, which is the phase space of the lit-only $\sigma$-game on $G$.

We determine the reachability relation for the digraph $\Gamma$. A surprising corollary of this work is that, for $\alpha, \beta \in \mathbf{F}_2^V$, basically, $\alpha$ can reach $\beta$ in $\Gamma$ if and only if $\alpha - \beta$ lies in the binary linear subspace spanned by $\{\alpha_v : v \in V\}$.

An important step of our work is to define the line graph of a multigraph and to provide a forbidden subgraph characterization. If the graph $G$ is loopless, as an application of our knowledge of the corresponding digraph $\Gamma$, we are able to determine the multiplicative group generated by $\{\mathcal{T}_v : v \in V\}$. We also indicate possible approaches on extending the work here to the general case of $G$ being a digraph.