

Let  $G$  be a graph. For any family  $R = R_k$  consisting of  $k$  pairs of vertices  $(s_1, t_1), \dots, (s_k, t_k) \in V(G) \times V(G)$ , a *path system* of  $G$  rooted at  $R$  is a family  $P_1, \dots, P_k$  of  $k$  paths such that  $P_i$  is an  $s_i, t_i$ -path for each  $i \in \{1, \dots, k\}$ , and  $V(P_i) \cap V(P_j) = \{s_i, t_i\} \cap \{s_j, t_j\}$  holds for any two distinct indices  $i, j \in \{1, \dots, k\}$ . When all  $(s_i, t_i)$  coincide with  $(s, t)$ , the path system is termed as an  $s, t$ - $k$ -rail. A path system is *spanning* if every vertex of  $G$  appears in at least one path in the system. A vertex ordering  $v_1, \dots, v_n$  of  $G$  is  $k$ -thick provided  $|\{j : v_j v_i \in E(G), i < j \leq n\}| \geq \min(k, n - i)$  holds for each  $i \in \{1, \dots, n\}$  and is *Hamiltonian  $k$ -thick* if it is  $k$ -thick and even corresponds to a Hamiltonian path of  $G$ . Peng Li and Yaokun Wu recently initiated the study of the relationship between thick orderings and spanning path systems.

The existence of a (Hamiltonian)  $k$ -thick ordering in a graph guarantees that, even when “some” nodes are faulty, the surviving graph still has a spanning path system with various given roots of a size comparable to  $k$ .

Here is one specific result stated in more precise language. Take two nonnegative integers  $t$  and  $s$  with  $s + t \leq k$  and  $t \geq 2$ . Let  $v_1, \dots, v_n$  be a Hamiltonian  $k$ -thick ordering of  $G$ . Then for every  $S \in \binom{V(G) \setminus \{v_1, v_n\}}{s}$ ,  $G - S$  contains a spanning  $v_1, v_n$ - $t$ -rail and such a  $t$ -rail can be found in linear time.